Towards Dynamically Consistent Real-time Gait Pattern Generation for Full-size Humanoid Robots

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Abstract—We propose a two-stage gait pattern generation scheme for the full-scale humanoid robots, that considers the dynamics of the system throughout the process. The first stage is responsible for generating semi-dynamically consistent step position and step time information, while the second stage is responsible for generation of gait pattern that is feasible and stable on the full-scale multi-degree-of-freedom humanoid robot. The approach allows for very rapid gait pattern regeneration during locomotion and includes information about present dynamic state when regenerating the new pattern. The paper contains description of a developed method, as well as experimental results proving its effectiveness.

Index Terms—bipedal locomotion, humanoid robots, natural dynamics, real-time control, pattern generation, LIP, preview control, ZMP

I. INTRODUCTION

For the humanoid robots to be useful for the society they have to be able to easily navigate in the human daily-life environment, which on one hand comprises a variety of surfaces that are difficult to model, while on the other hand is very dynamic and full of moving objects. To be able to handle this kind of environments the robot has to be equipped with the real-time control algorithms that are able to very quickly respond to the dynamic changes in the environment or to the changes in the motion that result from the inaccuracies in the models.

There is a number of existing real-time gait pattern generators for full-size humanoid robots able to quickly re-plan the gait pattern. Harada et al. developed and Morisawa et al. improved a method which given footsteps position analytically calculates the Zero Moment Point (ZMP) and Centre of Mass (CoM) trajectories providing a stable motion [1], [2]. However, the sudden change in the step length or inappropriate change in step time may result in the ZMP going out of the polygon of support. Urata et al. proposed a method for very dynamic disturbance rejection realized with use of optimization technique to find a foot placement necessary to dump the motion caused by an applied disturbance [3].

The method is however applied only to the standing robot and does not elaborate on the problem of rapid pattern regeneration during locomotion. Nishiwaki at al. developed a real-time method allowing for regeneration of pattern at an arbitrary point by changing ZMP reference [4]. This method employs a simple dynamic model to re-plan the length of the step and associated change in step time, it however, does not consider the effect a change of step time will have on the frontal plane motion.

We observed that in most of the real-time gait pattern generators the step position is generated by disregarding the dynamics of the system and focusing on the shape and structure (e.g. presence of obstacles) of the environment. The pattern generator later takes care of generating motion of CoM that will result in stable gait. While being very effective in the environment cluttered with obstacles, it limits capabilities of the rapid gait pattern regeneration. Also in many approaches the lateral and frontal plane of the motion are considered separately. In case of the very dynamic motion regeneration in the middle of a step these cannot be separated. In this paper we propose a real-time gait pattern generator for full-size humanoid robots, which is able to rapidly re-plan gait pattern by modifying a step position in sagittal and frontal plane, as well as a step time. We think that taking into account dynamics of the system when planning the footsteps is crucial for rapid gait pattern regeneration, therefore we propose the two step semi-dynamically consistent approach. In the first step we use single mass dynamic model to calculate step parameters, which in the second stage are used as an input to preview control based gait pattern generator [5] providing motion reference for execution on the full-size humanoid robot. In
our previous work we shown an off-line method [6], while in this work we focus on it’s online implementation.

We start our paper from a short overview of the whole system. In chapter III we explain the simple model used in the first stage of the pattern generation and follow it with explanation of the gait pattern generator. In chapter V we present a pattern generated online with the proposed method and results from experiment performed on our robotic platform WABIAN-2R (Fig. 1-a).

II. OVERVIEW OF DEVELOPED GAIT PATTERN GENERATOR

In our approach we propose a two-stage pattern generation method that considers the system dynamics throughout the process (See (Fig. 1-b)). The first stage comprises a simple model of a humanoid robot that allows for semi-dynamically consistent planning of feet placement and step time. We call it semi-dynamic because the present model assumes no double support phase. This stage takes as an input a desired average CoM forward velocity and calculates the motion in sagittal plane necessary to realize the reference. This provides information about the foot step length and step time. Then we use the step time information to calculate a feet placement in frontal plane necessary to constrain and control the CoM lateral sway. This way we generate the feet placement in sagittal and frontal plane, and the step time. This information is later passed to the second stage of the pattern generator which produces references of ZMP and feet trajectory and uses the preview controller to generate gait pattern feasible and stable when executed on full-size humanoid robot.

The advantage of this method when used in real-time gait pattern generation is that at both stages we can start the regeneration process from arbitrary point of swing phase and take it as source of initial conditions, thus fully accounting for the dynamic state of the robot at the regeneration point.

In the following sections we introduce the consecutive stages of the process. The model and methodologies used inside serve as an example and can be modified and developed to account for different factors of motion dynamics or shape and properties of the environment.

III. SEMI-DYNAMICALLY CONSISTENT STEP PLANNING

In the first stage of our pattern generator we focus on planning of feet placement and step time. For this purpose we use a Linear Inverted Pendulum model [7] (LIP - see Fig. 2-a) and derive equations necessary to suit our purpose. We chose the LIP because the motion equations are linear and allow simple manipulation yet the output approximates the tendency of the system’s natural dynamics. The assumptions used in the derivation of LIP motion equations are as follows:

- The whole system is represented by a single mass inverted pendulum with the mass placed at the height of the robot’s CoM in the free standing configuration.
- The CoM motion is constrained to the horizontal plane (the height of CoM is constant).

Fig. 2. a) LIP model in sagittal plane with definition of apexes, \( x_{\text{acc}} \) and \( x_{\text{dec}} \) in consecutive steps. b) CoM trajectory in frontal plane with respect to time. The blue lines denote step position, while the blue circles denote the moment of the foot exchange in LIP model. \( w_y \) denotes frontal plane position of CoM expressed in world coordinate frame.

- There is no torque acting between the system and the ground contact point.
- The friction is big enough to prevent slipping of the foot.

Thanks to the assumptions the motion in sagittal and frontal plane can be considered separately (decoupled) and expressed with the following linear equation:

\[
\ddot{x} = \frac{g}{z} x, \tag{1}
\]

where, \( z \) is the height of the CoM, \( g \) is the gravitational acceleration and \( x \) is the position of CoM in the foot coordinate frame. The foot coordinate frame is attached to the sole of the foot at the projection point of the ankle joint, with initial orientation aligned with the world coordinate frame. In this paper we use \( y \) to denote position of CoM in frontal plane and \( x \) to express the position in the sagittal plane. Both are expressed with respect to the current ground contact point of the inverted pendulum (foot coordinate frame). It is worth noting that the motion does not depend on the mass of the robot, but only on the height of the CoM.

The same model is used to derive a so-called Orbital Energy [8]. It expresses the kinetic and potential energy of the unit mass inverted pendulum. It is calculated by integrating (1) over the traveled distance and has the following form:

\[
E = \frac{1}{2} \dot{x}^2 - \frac{1}{2} \frac{g}{z} x^2 = \text{const}. \tag{2}
\]

where, \( x \) and \( \dot{x} \) are the position and velocity of the CoM at any point of single support phase.

In the following subsections we describe the motion equations necessary to plan the foot placement and its timing in order to achieve the desired CoM motion. We also show how do we start the motion of the system and elaborate on the real-time regeneration of the motion.

A. Generation of Motion in Sagittal and Frontal Plane

In this subsection we describe how using (1) and (2) we generated the reference foot steps to achieve the desired CoM motion.
First we start from defining the equivalent of apex in case of the LIP model (see Fig. 2). We define apex to be the point along the CoM trajectory in the considered plane of motion where CoM reaches the lowest absolute speed. In case when the CoM passes over the support foot (sagittal plane) the apex will be placed above the foot. In case when the CoM does not reach the support foot and starts falling back (frontal plane), apex will be the point where CoM velocity reaches 0.

1) Sagittal plane: Taking a closer look at the trajectory of CoM during the single support phase, we can notice that the CoM decelerates before reaching the apex and accelerates after passing the apex point. By controlling the distance of acceleration during one step and deceleration during the consecutive step we can control the change of velocity of CoM in the consecutive apexes. We can derive the equation governing the relation between the two distances and the step length from the Orbital Energy equation (2). For derivation purposes we define the desired step length as $x_{sl}$ and assume that it is given. We also define the ratio between the CoM in the consecutive apexes as:

$$r = \frac{x_{acc}}{x_{sl}}.$$ (3)

The following equation expresses CoM velocity in apex $i + 1$:

$$i+1\dot{x}_{apex} = i\dot{x}_{apex} + \frac{g}{2}(x_{acc} - i+1x_{dec}),$$ (4)

where, the left superscript denotes the step number, this notation is used from here on. By noting that $i\dot{x}_{sl} = i\dot{x}_{acc} + i+1\dot{x}_{dec}$, substituting (3) into (4) and simplifying, we obtain the following equation:

$$r = \frac{1}{2} \frac{i+1\dot{x}_{apex} - i\dot{x}_{apex}}{i\dot{x}_{sl}} + \frac{1}{2}.$$ (5)

Given the step length, (3) and (5) we can calculate the acceleration and deceleration distances. The acceleration distance determines the time remaining to the foot exchange from the previous apex, while the deceleration distance determines the distance between the CoM and the foot position during the foot exchange. Repeating the calculation for each consecutive step, given its apex reference velocity and desired step length, we are able to calculate the motion of CoM, foot steps and the single support time in the sagittal plane.

2) Frontal plane: The motion in frontal plane, unlike sagittal, comprises of CoM swings that should never cross the support foot position (Fig. 2-b), unless the robot is subjected to very high disturbances. Planing motion in frontal plane is reduced to determining the step position so that the CoM does not cross the foot and the single support time has the desired duration.

By starting the motion generation from the sagittal plane, we predefine the single support time for each step of the gait. Thus, to plan the frontal plane CoM trajectory of each step we need to find such foot placement that will result in the same single support time as in sagittal plane. Apart from the step time, we also want to be able to control the step width as that affects the lateral sway of CoM and is constrained by the movable range of the robot. One last parameter that we want to control is the position of CoM in the world coordinate frame during the foot exchange, since this will let us to control the locomotion trajectory of the robot Fig. 2-b. In derivations of necessary formulas we use equations of motion obtained by integration of (1)

$$y(t) = yo\cosh\left(\frac{t}{k}\right) + \frac{1}{k}\sinh\left(\frac{t}{k}\right),$$ (6)

$$\ddot{y}(t) = \frac{1}{k}\sinh\left(\frac{t}{k}\right) + yo\cosh\left(\frac{t}{k}\right),$$ (7)

where, $y_0$ and $\dot{y}_0$ are the initial position and velocity of CoM with respect to the support foot, $t$ is the time that elapsed from the initial configuration, $k = \sqrt{\frac{y}{g}}$.

Assuming that the apex velocity reference in sagittal plane is constant over a number of steps, the single support time $(T_{ss})$ during these steps will also be constant and the system will be in a limit cycle. Now, given arbitrary initial conditions we would like to bring our LIP model into the limit cycle in the frontal plane, in which the initial CoM position $y_0$ at the moment of foot exchange is equal to the half of the step width and the CoM position in the world coordinate frame is equal to $w\dot{y}_0$ (here, the left subscript denotes that the value is expressed in the world coordinate frame). The initial velocity should be determined so that after the time $T_{ss}$ elapses, the CoM will be in exactly same position $y_0$. To calculate the initial velocity we can use formula (7) knowing that for the above to be true the velocity after $T_{ss}/2$ should be zero.

$$i\dot{y}_0 = -iyo\frac{k\sinh(iT_{ss}/2k)}{\cosh(iT_{ss}/2k)},$$ (8)

where, $iT_{ss}$ denotes single support time in step i. In order to achieve this velocity and to assure that the CoM position in the world coordinate frame during foot exchange is at the desired place, when starting from arbitrary initial conditions, we need to control the step position of two preceding steps. By combining (6) and (7) for two preceding steps and adding condition for CoM being at the desired global position during foot exchange we arrive at the following set of equations.

$$i+1y(i+1T_{ss}) = i+1y(0) + i+1A + i+1\dot{y}(0) + i+1B,$$

$$i+1\dot{y}(i+1T_{ss}) = i+1y(0) + i+1C + i+1\dot{y}(0) + i+1A,$$

$$i+y(T_{ss}) = i+y(0) \cdot C + i\dot{y}(0) \cdot A + i+1\dot{y}(0),$$ (9)

$$i+y(0) - w\dot{y}(0) = -i+y(0) + i+y(T_{ss}) - i+y(0) + i+1y(i+1T_{ss})$$

where, $\dot{A} = \cosh(iT_{ss}), \dot{B} = k \cdot \sinh(iT_{ss})/k$ are elements of equation (6) and (7). The simplification yields a set of linear equations, whose solution provides the desired step position in two consecutive steps preceding the step of interest (here step number $i + 2$).

$$\begin{bmatrix}
   i+y(0) \\
   i+y(0) \\
   i+y(T_{ss}) \\
   i+y(T_{ss}) \\
   i+y(i+1T_{ss}) \\
   i+y(i+1T_{ss})
\end{bmatrix} = \begin{bmatrix}
   iC + i+1B + iA - 1 \\
   iC + i+1A - 1 \\
   iC + i+1B + iA \\
   iC + i+1A \\
   iC + i+1B \\
   iC + i+1A
\end{bmatrix}^{-1} \times \begin{bmatrix}
   i+y(0) \\
   i+y(0) \\
   -i+y(0) - i+y(0) + i+y(T_{ss}) - i+y(0) + i+y(i+1T_{ss})
\end{bmatrix}$$ (10)
We use the above equations to calculate the step position in every single step calculation. The method proved to be stable in a variety of tested scenarios. Since the above equations do not include the system limitations, like feasible workspace, the result has to be verified against them and if necessary limited. Since the step position is recalculated at each foot exchange based on the current state, even after limiting the step position the robot will reach the desired global position in the following steps.

B. Starting the Motion

Since the methods described in III-A assume that the system is already in motion, we need to put it into motion by specifying initial CoM trajectory.

1) Sagittal plane: We initiate the motion by applying the virtual torque to the ankle joint until the mass reaches the desired \( x_{\text{ave}} \). The trajectory of the CoM is calculated with modified version of (1) to include the effect of the torque.

\[
\ddot{x} = \frac{g}{2} x + \frac{T_y}{m z}
\]  

(11)

2) Frontal plane: We start from generating initial swing in the direction of the first swing foot. The swing is calculated in a way that will make the CoM reach the desired velocity when crossing the sagittal plane (middle) on the way back towards the first support foot. The desired velocity is calculated so that the time needed for CoM to return back to the sagittal plane is equal to \( T_{ss} \).

C. Real-time Regeneration

The described method can be applied to regenerate again pattern from arbitrary point of the swing phase. The point where the new trajectory is going to be connected serves as a source of initial conditions of both the LIP model in the first stage of the pattern generation and the Preview Controller in the second. In this section we describe the changes and considerations that need to be taken into account when regenerating the pattern.

Since the regeneration of the pattern starts when the robot is in motion there is no need to generate the starting motion and we can proceed to planning motion as described in III-A. Inside the first stage we calculate each step individually, so the only difference between the algorithm described in III-A is the calculation of the first step. The initial conditions for the first step, namely the position and velocity of CoM with respect to the support foot, are taken from the reference calculated in the first stage of the original pattern generation.

1) Sagittal plane: For generation of the sagittal plane motion we can use (6), (7) and (5). Since the starting point is not an apex we need to modify (5) used to calculate ratio between the acceleration and deceleration to include the starting position and velocity of the CoM.

\[
r = \frac{1}{2} \left( \frac{z + \frac{1}{g} x^2_{\text{apex}} - \frac{z_0^2}{g} + \frac{x_0^2}{x_{st}^2}}{2} + \frac{1}{2} \right)
\]  

(12)

where, \( x_0 \) is the position and \( x_0 \) is the velocity of CoM at the pattern connection point. We have to ensure that the result fulfills \( r > \text{abs}(\frac{2x}{z_0}) \), since the acceleration distance cannot be smaller than the initial CoM position.

2) Frontal plane: The generation of motion in the frontal plane can remain the same, since the initial conditions used in the calculation of the step position in frontal plane are taken from the expected foot exchange point. As we regenerate the motion always starting from the foot exchange point, knowing the current conditions and the remaining duration of the single support phase we can calculate conditions during the foot exchange with (6) and (7). The example of pattern generated online with multiple regenerations is presented in chapter V.

IV. GAIT PATTERN GENERATION

A gait pattern is a set of references which fully define the robot’s motion in 3D space. Usually these are the task space trajectories of feet, hands and waist which later with use of inverse kinematics are transformed into the joint angle references. The objective of a gait pattern generator is to provide the gait pattern which if executed on the full-scale humanoid robot will result in stable locomotion. It’s main task is to calculate the CoM or waist trajectory which result in a stable locomotion.

In this stage, based on the footsteps and step timing we define the feet transition trajectories with use of the fifth-order polynomials. Based on the preliminary CoM trajectory we calculate the reference ZMP trajectory to be used by gait pattern generator [6]. In our application we decided to use a Preview Control based pattern generator [5]. Because of it’s simplicity, inclusion of multi-body dynamics system (MBS) in the process and possibility of starting from arbitrary condition of CoM.

A. Preview Control

We do not intend to present the derivation and details of the Preview Control, instead we try to explain its basic working principles. Readers interested in details of the Preview Controller should refer to [5].

The task of the preview controller is, given the ZMP reference, to generate the CoM (or waist)trajectory of the robot that will result in the ZMP resulting from MBS simulation matching the reference ZMP with the specified error. The input comprises ZMP trajectory and the reference trajectories which pre-define the motion of the robot in 3D space. In our case, these are the feet and hands trajectories, the waist trajectory is determined and later modified by the preview controller. The flow-chart of the preview control is presented on Fig. 3. In the first iteration of the Preview Control the initial waist trajectory in sagittal and frontal plane separately is calculated with the state space equation of which the controlled variable is expressed with the following equation:

\[
u(i) = -G_i \sum_{j=0}^{i} e(j) - G_z x(i) - \sum_{j=0}^{N_L} G_p(j) ZMP_{x}^{e_f}(i+j),
\]  

(13)

where, \( G_i \) is an integral gain trying to compensate for the steady state ZMP error, \( G_z \) is a matrix of proportional gains serving as a state feedback and \( G_p \) is a vector of preview gains which modify the control variable depending on the
future ZMP reference, \( e(i) \) is an error between the reference ZMP and the ZMP calculated with MBS model in the \( i^{th} \) sample of the trajectory, \( x \) is the state vector containing position, velocity and acceleration of waist in sagittal plane, \( N_s \) is the number of preview samples and \( ZMP_{prev} \) is the ZMP reference trajectory. This iteration provides the initial waist trajectory and the ZMP error between the original reference and the MBS model. In the following iterations of the Preview Controller the waist trajectory is modified based on the ZMP error from the previous iterations, according to the following equation

\[
u(i) = -G_i \sum_{j=0}^{i} e(j) - G_{\Delta x} x(i) - \sum_{j=1}^{N_s} G_{p}(j) e^{prev}(i + j), \tag{14}\]

where, \( \Delta x \) is the state space correction vector and \( e^{prev} \) is the ZMP error in the previous iteration. This iteration is repeated either until the ZMP error falls below a given threshold or given number of iterations. In our case, just two iterations were sufficient to reduce ZMP error below 0.003m for the regular forward walk.

The waist trajectory generated in the last iteration, together with remaining motion references become the final gait pattern of the generator.

B. Real-time Regeneration

The only necessary modification to the preview controller is the initial value of the state vector. In this case we use the value at the connection point, obtained from the Preview Controller when generating the original pattern.

V. EXPERIMENTS

In this section we describe the setup and results of experiment performed to validate the online implementation of the algorithm. The experiment was performed on our research platform WABIAN-2R (Fig. 1) [9]. The forward reference velocity was controlled with joystick. The range of its motion was projected to the forward velocity reference between 0 – 0.15m/s. The joystick was connected to laptop PC which was communicating with robot’s control computer through LAN (see Fig. 4).

During the experiment an operator was modifying velocity of locomotion by suddenly accelerating and decelerating the motion. Fig. 5 shows the cut out of the reference ZMP trajectory generated by the gait generator. The step length varies depending on the reference velocity. The initial reference step time used to calculate the apex velocity is set to 1s. It is however automatically adjusted depending on the changes in the reference velocity. It is best visible on Fig. 5 at the moment when the robot accelerates or decelerates. Fig. 6 presents the same section of CoM and feet trajectory. It is visible that the gait generator adjusts the step position in both sagittal and coronal plane depending on the desired reference velocity. The biggest changes are visible during motion acceleration, when the robot in order to gain on kinetic energy extends the acceleration distance and shortens the deceleration distance. This results in shortened step time in consecutive step and thus to contain the swing of the CoM in front of the robot requires bigger side step. Fig. 7 and Fig. 8 show the reference and measured value of ZMP in sagittal and frontal plane respectively. Apart from 15s when early ground contact occurred, there was no major deviation of the measured value from reference.

The experiments we performed showed that our gait generator is able to exploit major dynamic components affecting
forward locomotion, which are step timing and step placement. The measured ZMP data shown that resultant gait pattern is stable on full-size humanoid robot.

VI. SUMMARY

In this paper we presented a semi-dynamically consistent real-time gait pattern generator capable of dynamic pattern regeneration. The gait pattern generation process is divided into two stages. The first stage is based on the inverted pendulum model that allows for easy and autonomous generation of the step placement in sagittal and coronal plane as well as a step time. The model used to present the method is constrained to the forward locomotion on the flat ground, but it can be easily substituted with more advanced models including the ground contact and angular momentum effects or models enabling locomotion in 3D or in environments with different mechanical properties. The second stage is responsible for generation of the final gait pattern based on the information from the first stage. It uses a Preview Control method to fine tune the reference trajectory in order to ensure locomotion stability. We show that using the online implementation of the proposed scheme we are able to easily generate and regenerate gait pattern which is consistent with natural dynamics and stable when executed on full-size humanoid robot (for the better image of the motion we encourage the reader to view the video of the experiment [9]).

Since the method allows as for regeneration of the motion from arbitrary moment of the swing phase using the present dynamic state of the robot as initial conditions we believe that the method has a high potential of being applied to the disturbance rejection and for improvement of locomotion stability when walking on unstructured surfaces. These topics are currently in the scope of our future works. Also LIP model used in the first stage does not include the double support phase, what compromises the dynamic consistency and ability to regenerate the motion during the double support phase. We plan to eliminate this limitation in the near future.

ACKNOWLEDGMENT

This study was conducted as part of the Research Institute for Science and Engineering, Waseda University, and as part of the humanoid project at the Humanoid Robotics Institute, Waseda University. It was supported in part by JSPS KAKENHI (Grant Number: 24360099) and JSPS Institutional Program for Young Researcher Overseas Visits. It was also partially supported by SolidWorks Japan K.K, DYDEN Corporation and Cybernet Systems Co.,Ltd whom we thank for their financial and technical support.

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